

# Large-Eddy Simulation and Lagrangian Two-Particle Modeling of Mean and Fluctuating Concentrations in the Atmospheric Boundary Layer

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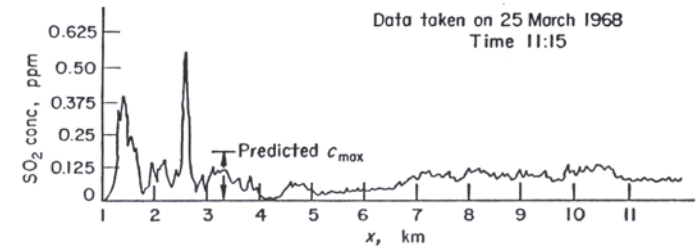
Collaborators: P.P. Sullivan, E.G. Patton, and A. Wyzogrodski

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and U.S. Army Research Office

For presentation at the MAC-MAQ Conference  
Davis, CA, 11 – 13 September, 2019

# Background

- Importance of statistical variability
  - Estimate peak concentrations—toxics, air quality, odors, chemistry, etc.
  - Need: mean, variance, probability distrib.
- Statistics for variable averaging time,  $T_{av}$ 
  - Requires “relative dispersion” of plume
- Approach
  - Lagrangian two-particle dispersion model (L2PDM) driven by large-eddy simulations (LES) of the convective boundary layer (CBL)
- Goal
  - Generate dispersion realizations & statistics for  $T_{av}$ 's: 40 s – 30 min
  - Demonstrate applicability of L2PDM to mean & fluctuating concentrations with convection tank data
  - Provide numerical data for testing simpler models

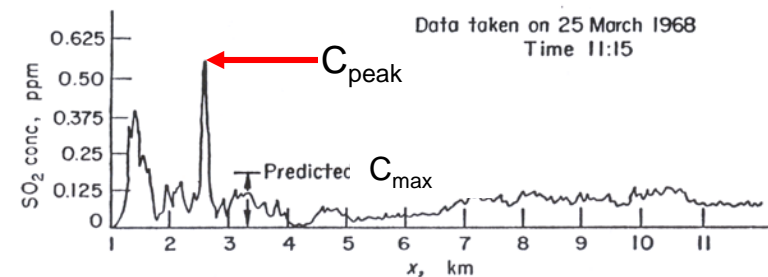


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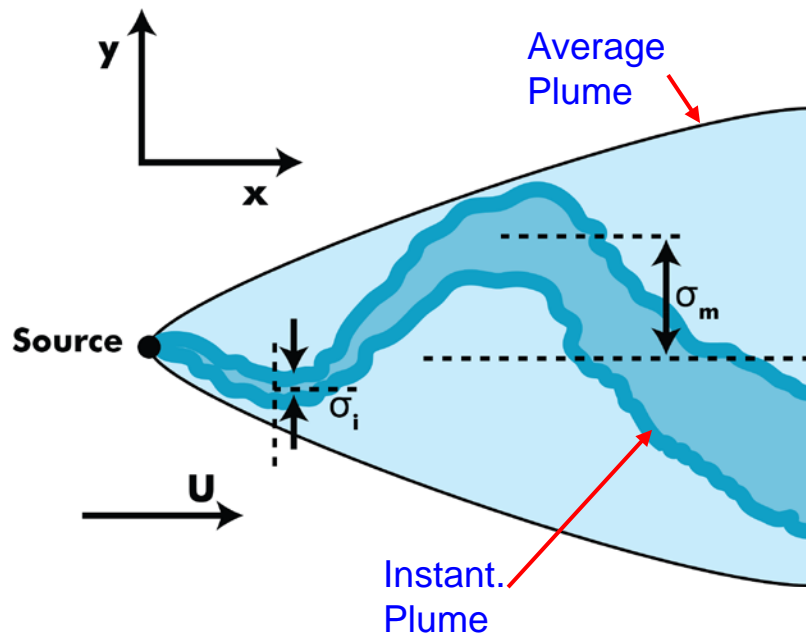


Figure 14. Keystone plume, May 25, 1968, 1047 EST.

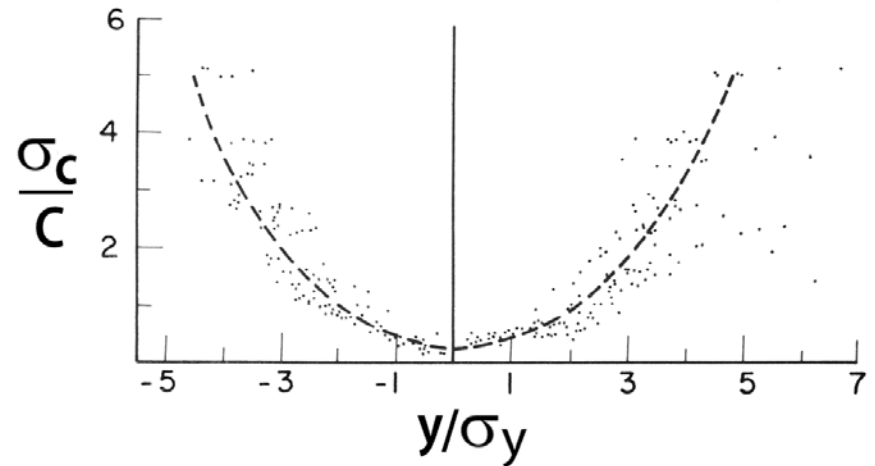


# Generation of Concentration Fluctuations

Meandering Plume Model  
(Gifford 1959)



Concentration Fluctuation Intensity  
(Csanady 1973)



$$\sigma_y = (\sigma_i^2 + \sigma_m^2)^{1/2}$$

**Short times**

$$\sigma_i^2 = \sigma_r^2 \propto \epsilon t^3 \quad t \ll T_L$$

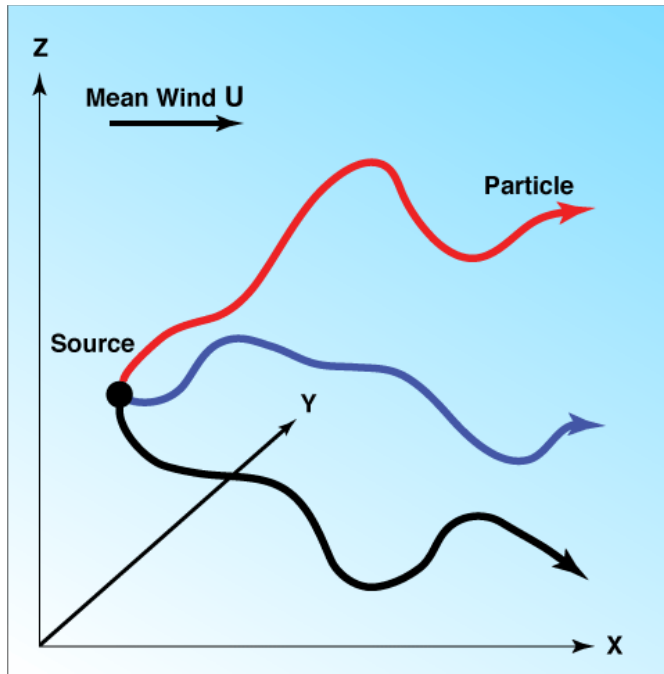
$$\sigma_m = \sigma_{vm} t$$

(Batchelor, 1950)

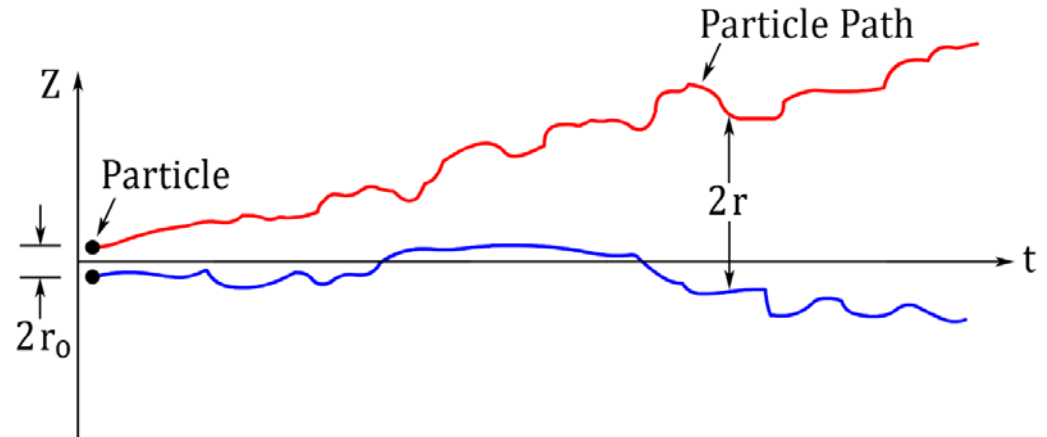
(Csanady, 1973)

# Lagrangian 1- & 2-Particle Dispersion Models

Applies to 1- & 2-particle models



2-particle model



$$v_1(\mathbf{x}_{01}, \mathbf{x}_{02}, t) = u_{R1}(\mathbf{x}_{p1}, t) + u_{S1}(\mathbf{x}_{p1}, \mathbf{x}_{p2}, t)$$

$$v_2(\mathbf{x}_{02}, \mathbf{x}_{01}, t) = u_{R2}(\mathbf{x}_{p2}, t) + u_{S2}(\mathbf{x}_{p1}, \mathbf{x}_{p2}, t)$$

$u_{Ri}$  = resolved LES velocity, particle  $i$

$u_{Si}$  = stochastic subgrid-scale (SGS) velocity

Adopt Thomson's (1990) stochastic model for  $u_{Si}$

Concentration

$$c(\mathbf{x}, t) = Q \int p_1(\mathbf{x}, t; \mathbf{x}_{S1}, t'_1) dt'_1$$

$\uparrow$   
 $t = nT_{av}$

# LPDMs Driven by LES

Large-Eddy Simulations (LES)  
(Moeng & Sullivan, 1994, NCAR LES;  
Prusa et al, 2008, EULAG )

- CBL Setup:
  - 5 km X 5 km X 2 km domain
  - 96<sup>3</sup> grid points
  - $z_i = 1000$  m,  $w_* = 2$  m/s,
  - $z_i/w_* = 500$  s,  $U = 3$  m/s,  $U/w_* = 1.5$
  - Highly convective:  $-z_i/L = 106$  (NCAR)  
= 78 (EULAG)
- 500 stored LES data files (NCAR)
  - at 10 s intervals
  - 210 files (EULAG)

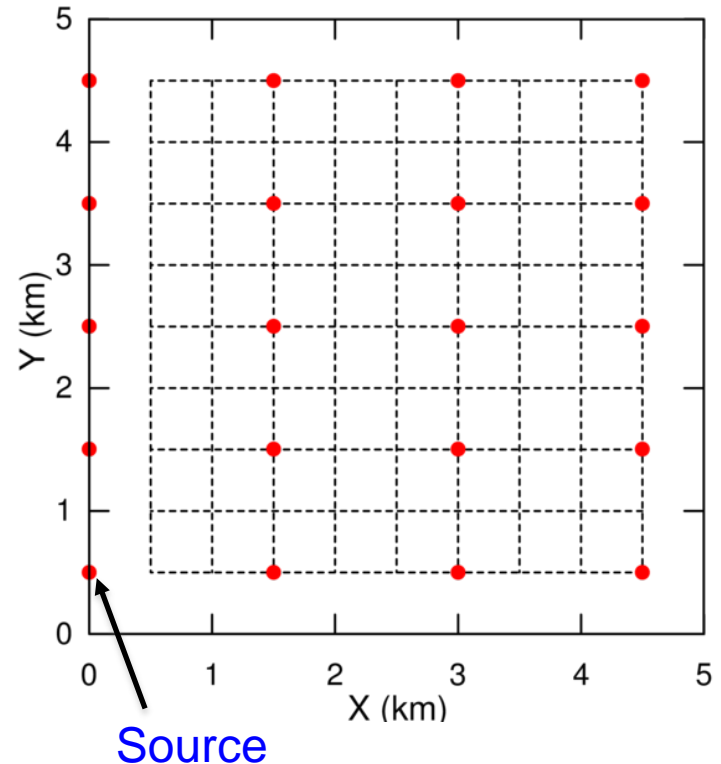
Subgrid TKE

$e_s$

Dissipation rate

$$\epsilon = c_e e_s^{3/2} / \ell$$

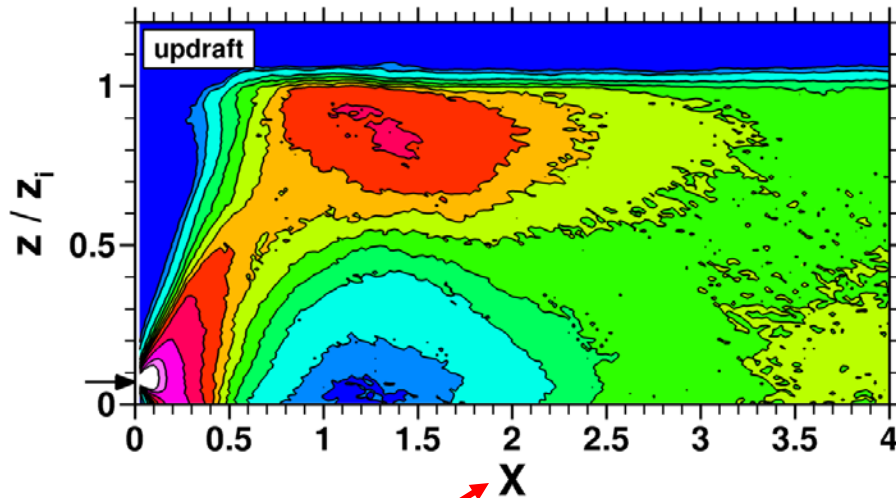
Source Grid



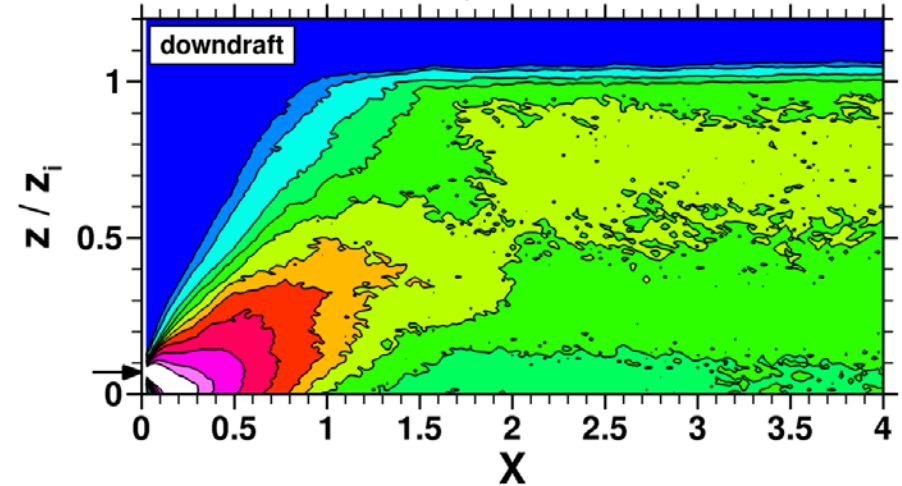
# Crosswind-Integrated Concentrations (CWIC, $C^y$ )

$z_s = 0.07 z_i$ ,  $T_{av} = 30$  min, L1PDM (Weil et al., 2012, BLM)

## Strong Updraft



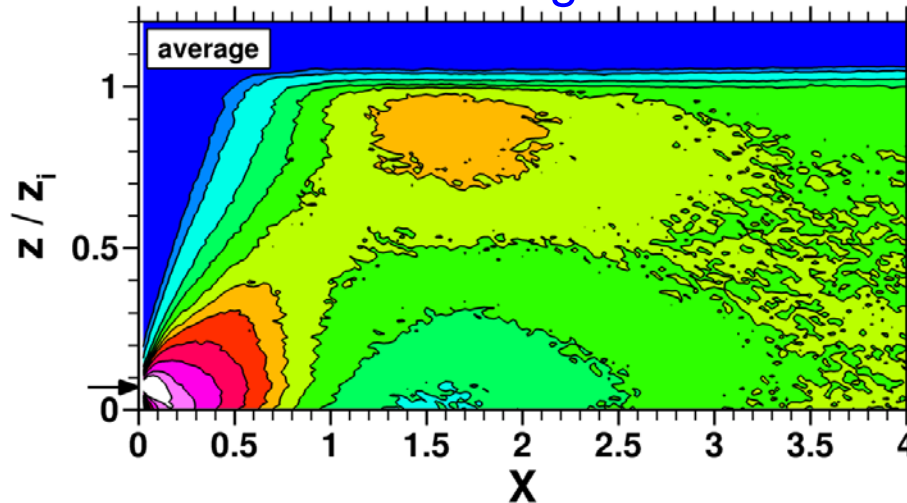
## Strong Downdraft



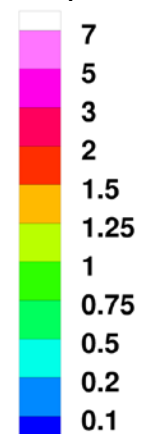
Convective scaling:

$$X = w_* x / (U z_i)$$

## Average

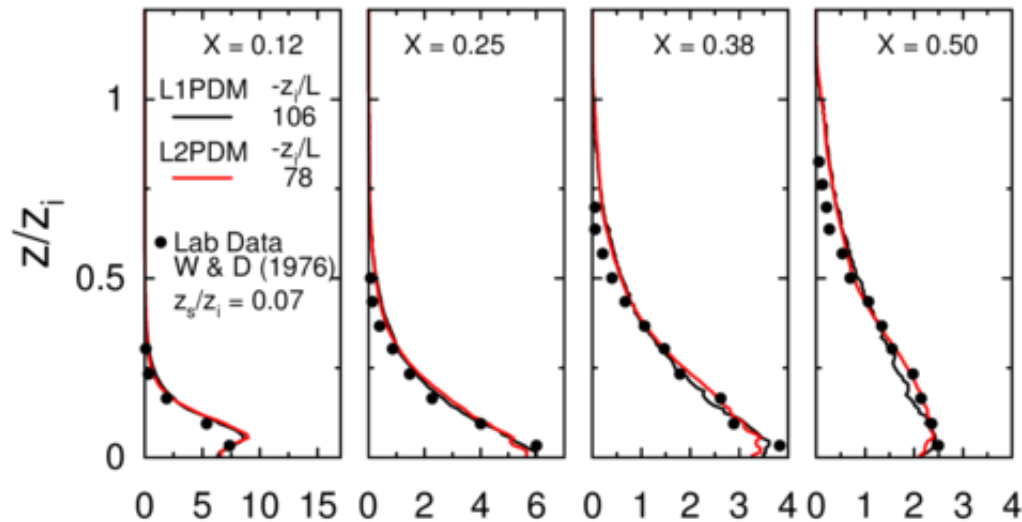


$C^y U z_i / Q$

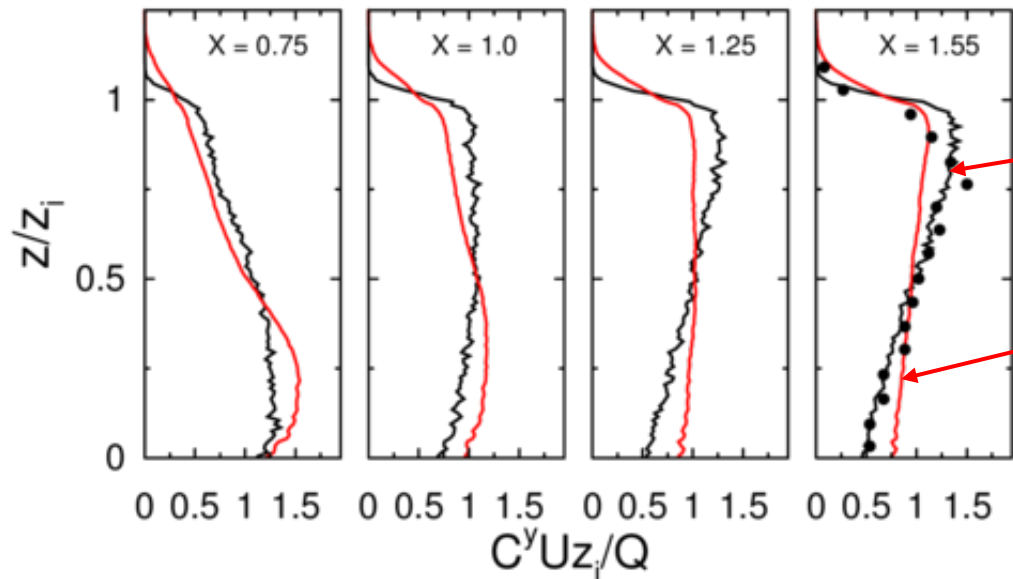


# Vertical Profiles of Mean CWIC

$z_s = 0.07 z_i$ ,  $T_{av} = 30$  min, L1PDM & L2PDM



$$X = w_* x / (U z_i)$$



1-particle; Weil et al. (2004)

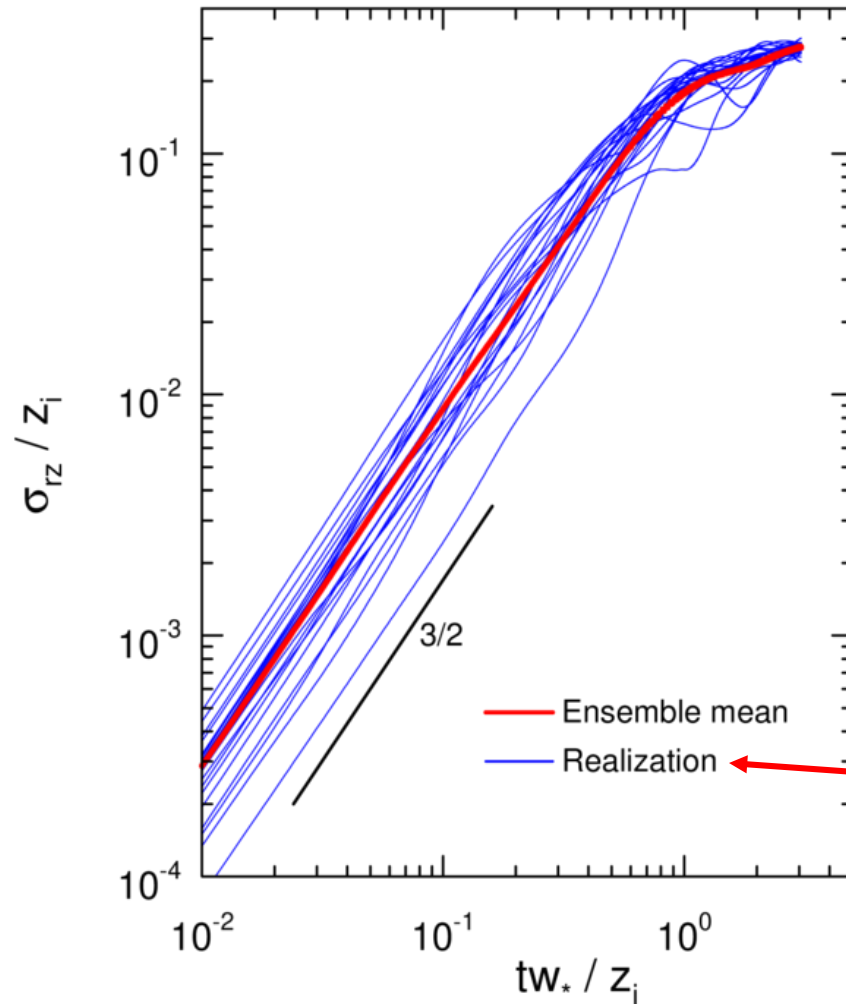
$-z_i / L = 106$

2-particle; Current work

$-z_i / L = 78$



# 2-Particle LPDM: Relative Dispersion



Batchelor (1950)

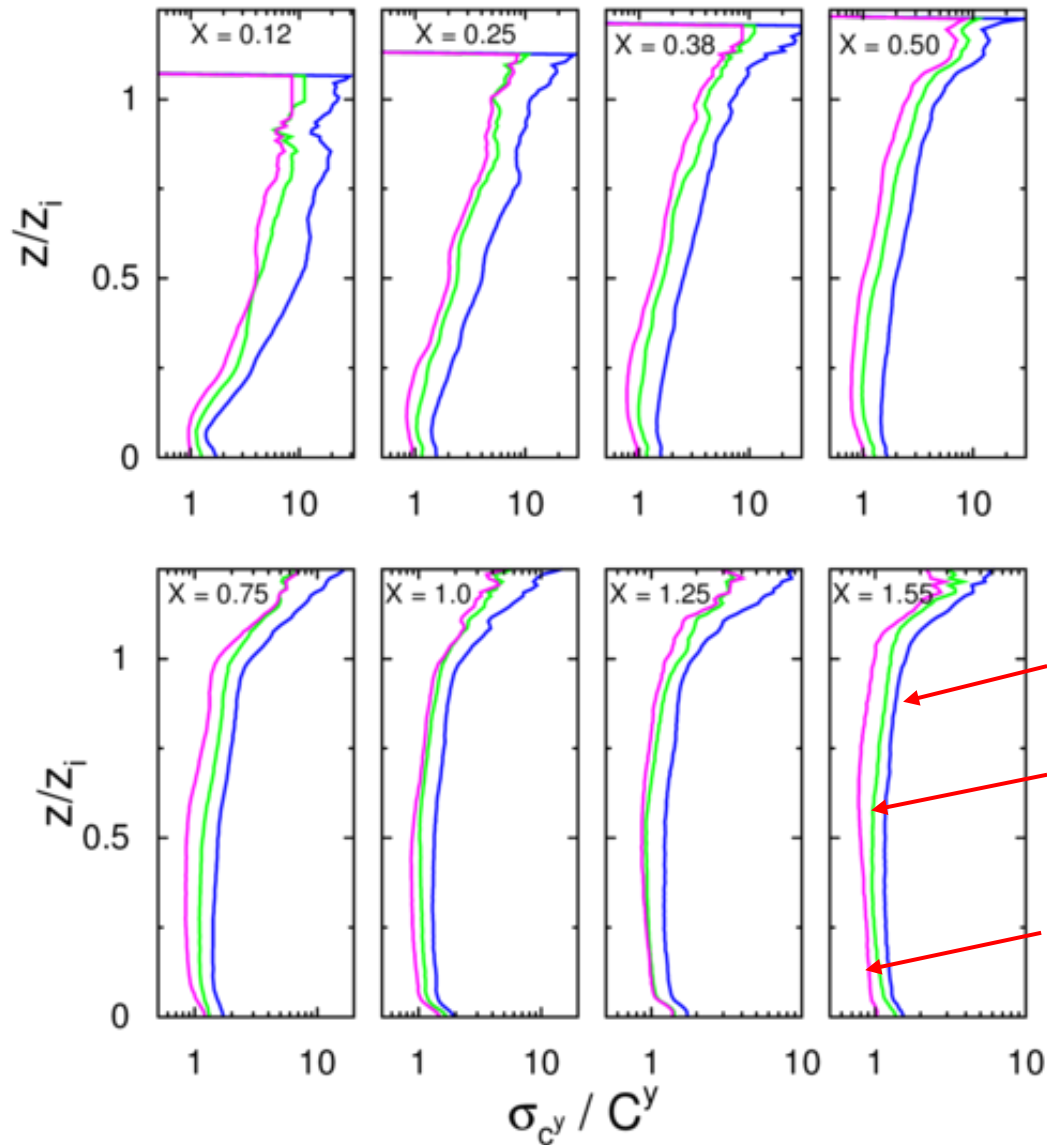
$$\sigma_r \propto \epsilon^{1/2} t^{3/2} \quad t \ll T_L$$

$$\sigma_r \propto \sigma_v (T_L t)^{1/2} \quad t \gg T_L$$

From 20 CBL sources

# Vertical Profiles of CWIC Fluctuation Intensity, $\sigma_{C^y} / C^y$

$z_s = 0.07z_i$  , L2PDM



Eddy turnover time

$$t_* = z_i / w_*$$

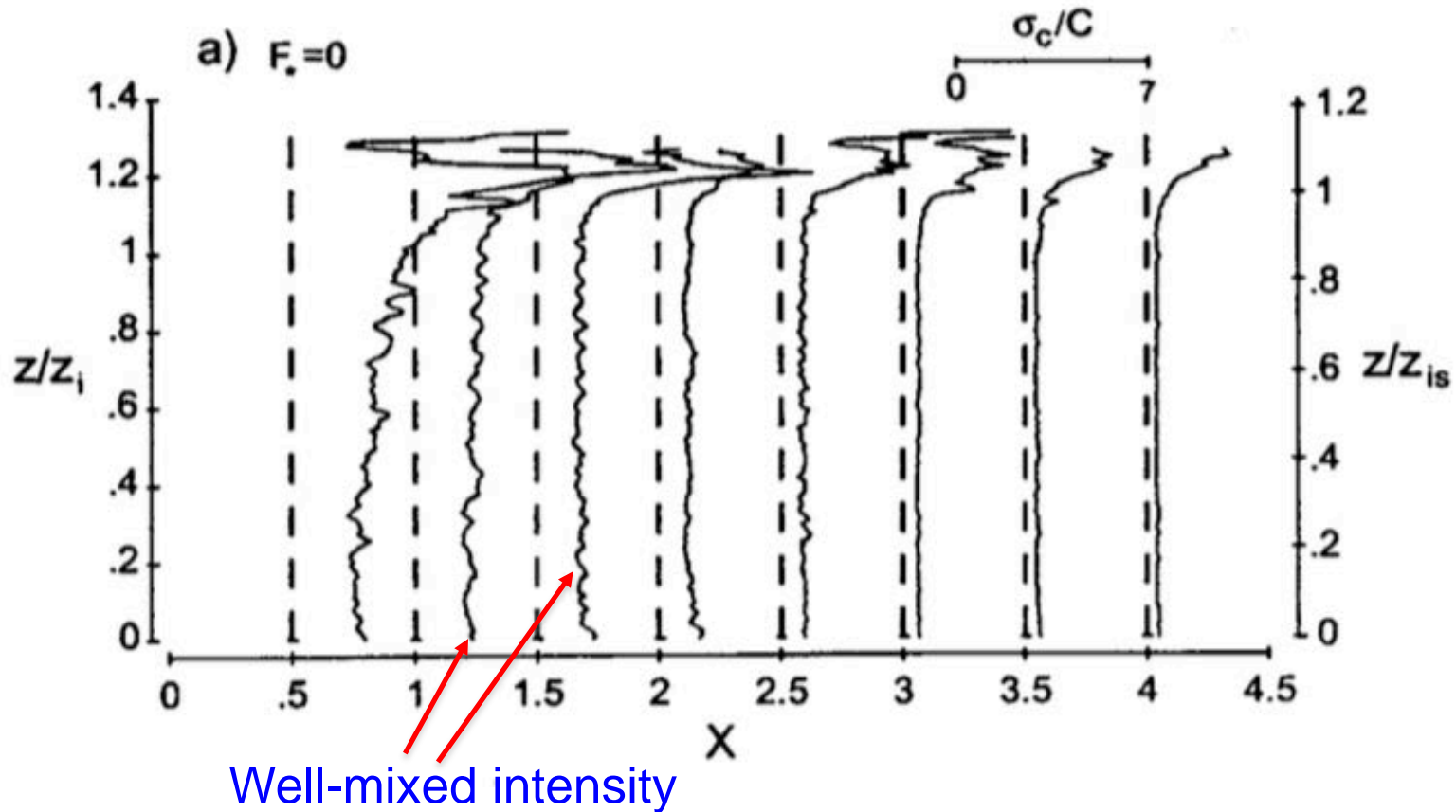
$T_{av} = 60 \text{ s}$  (= 0.12  $t_*$ )

420 s (= 0.84  $t_*$ )

700 s (= 1.36  $t_*$ )

# Vertical Profiles of Concentration Fluctuation Intensity, $\sigma_c/C$

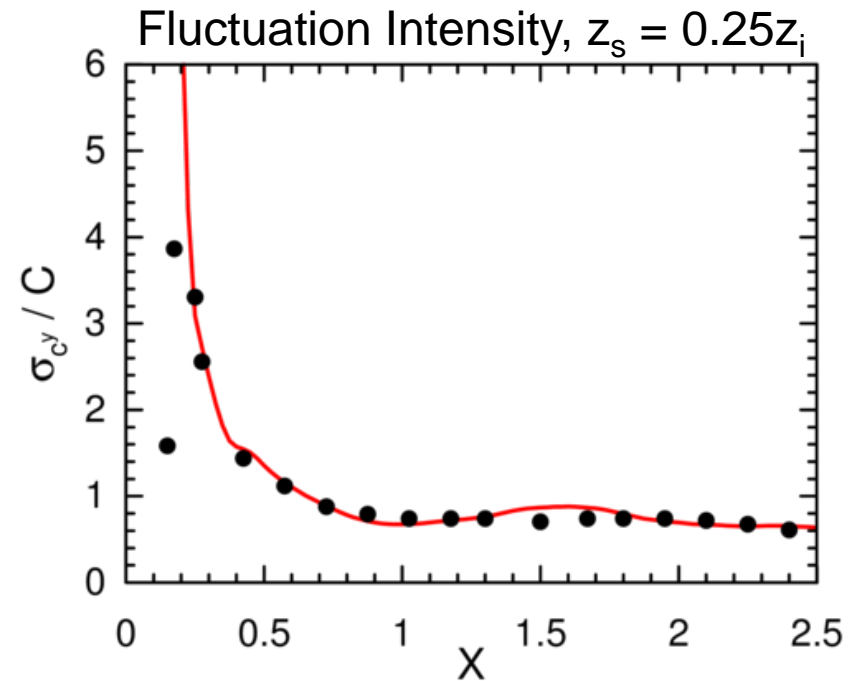
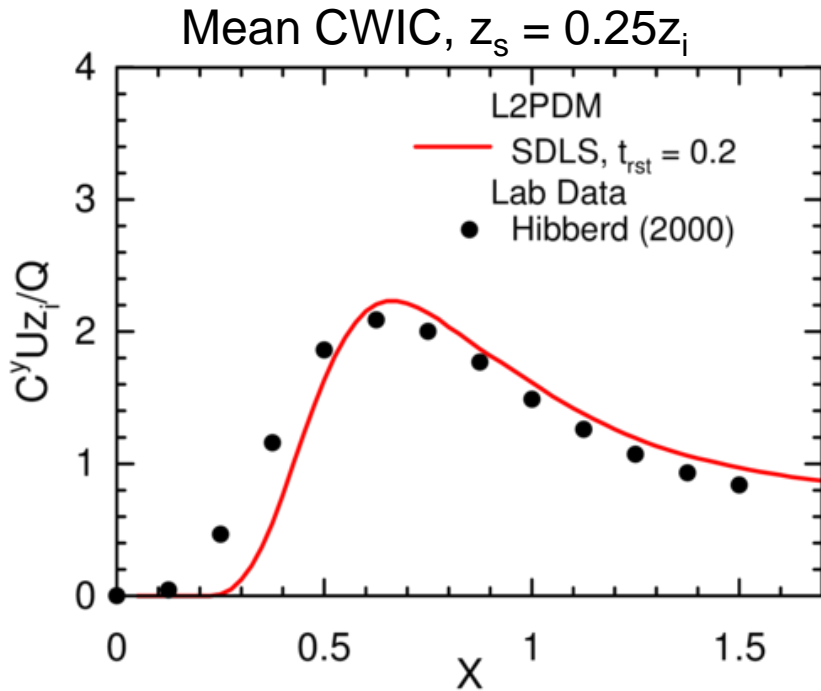
Convection Tank Data,  $z_s = 0.15z_i$  (Weil et al., 2002)



# Surface Mean CWIC & Fluctuation Intensity

Short Duration Line Source (SDLS),  $0 \leq t_r \leq 0.2t_*$

L2PDM vs Convection Tank Experiments (Hibberd, 2000)

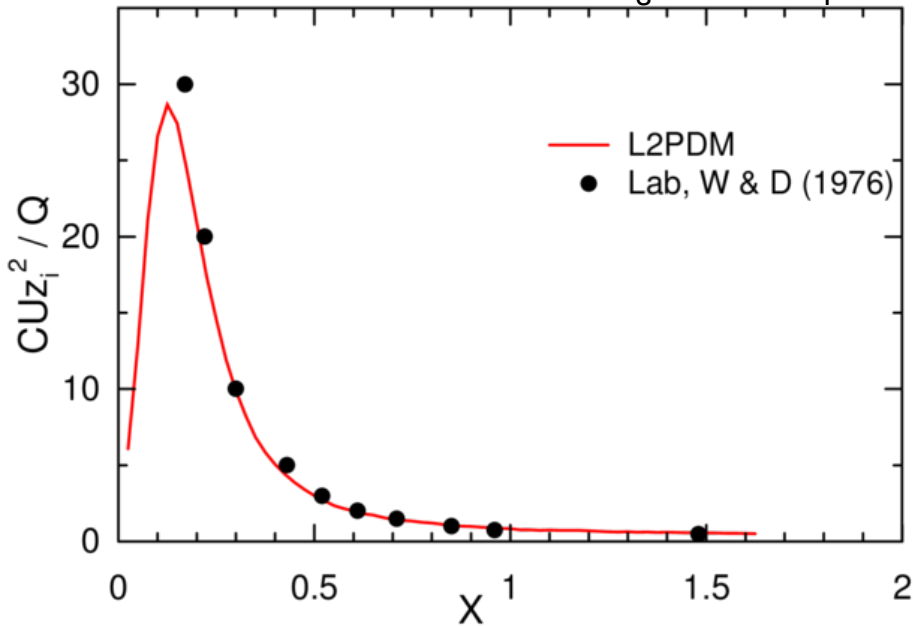


$T_{av} = 1 \text{ min}$

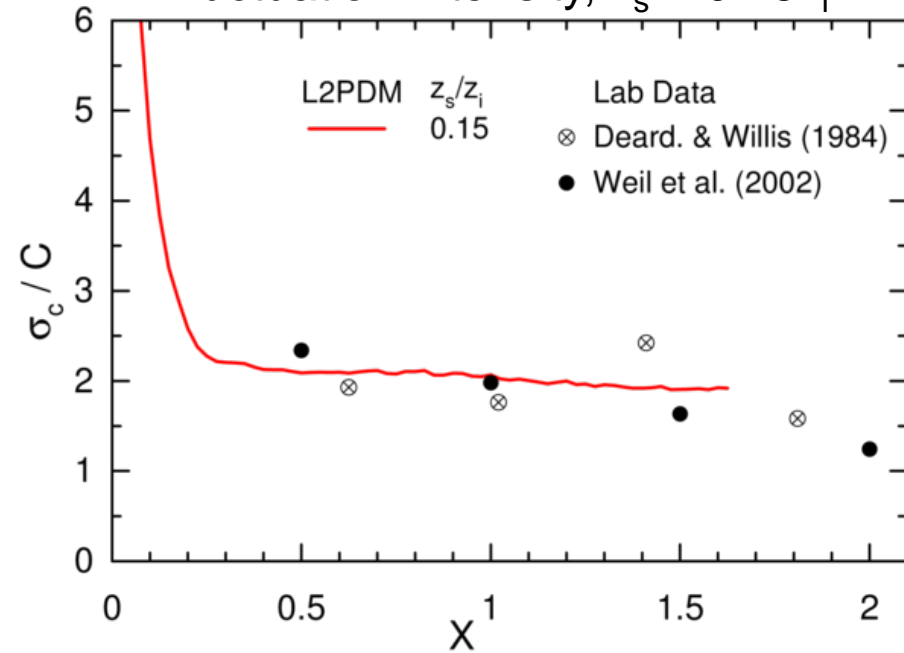
# Surface Mean Concentration & Fluctuation Intensity

Along plume centerline ( $y = 0$ )

Mean Concentration,  $z_s = 0.07z_i$



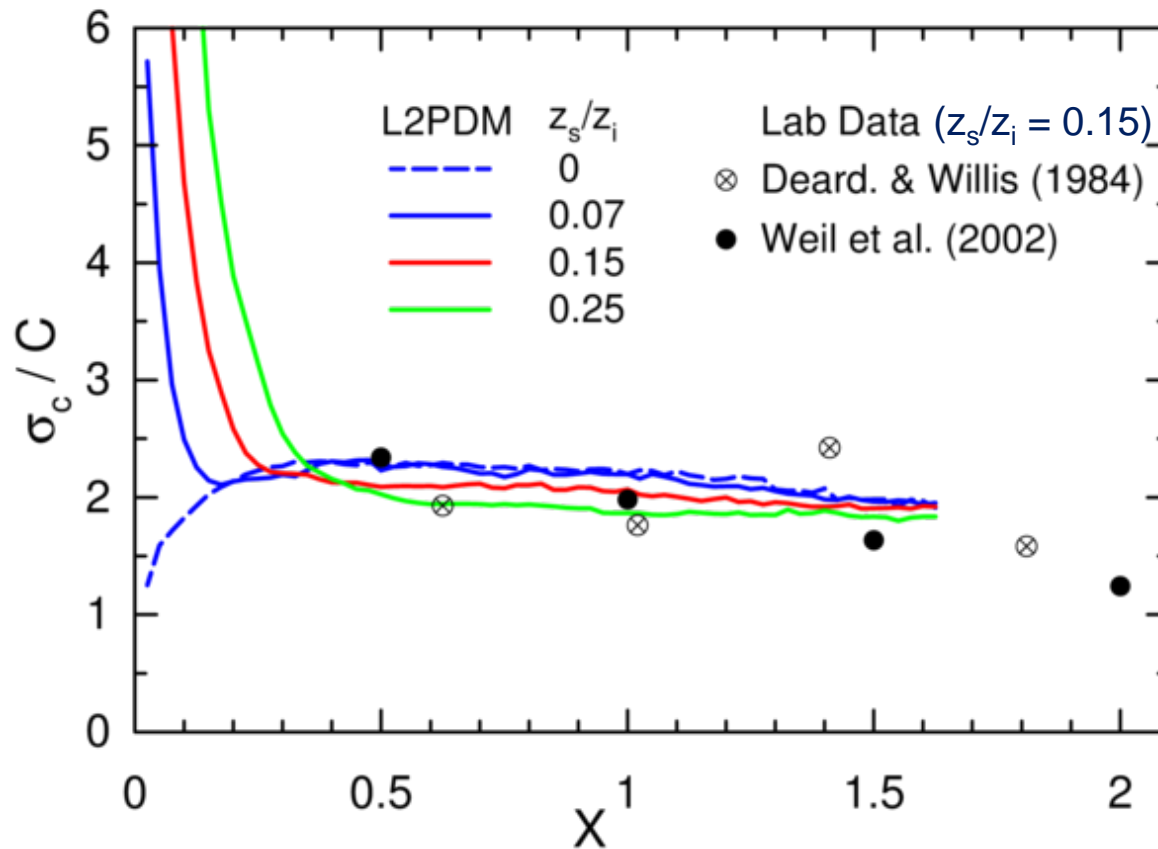
Fluctuation Intensity,  $z_s = 0.15z_i$



$$T_{av} = 1 \text{ min}$$

$$T_{av} = 0.12 t_*$$

# Concentration Fluctuation Intensity at Surface: L2PDM vs Lab Data, Multiple Source Heights



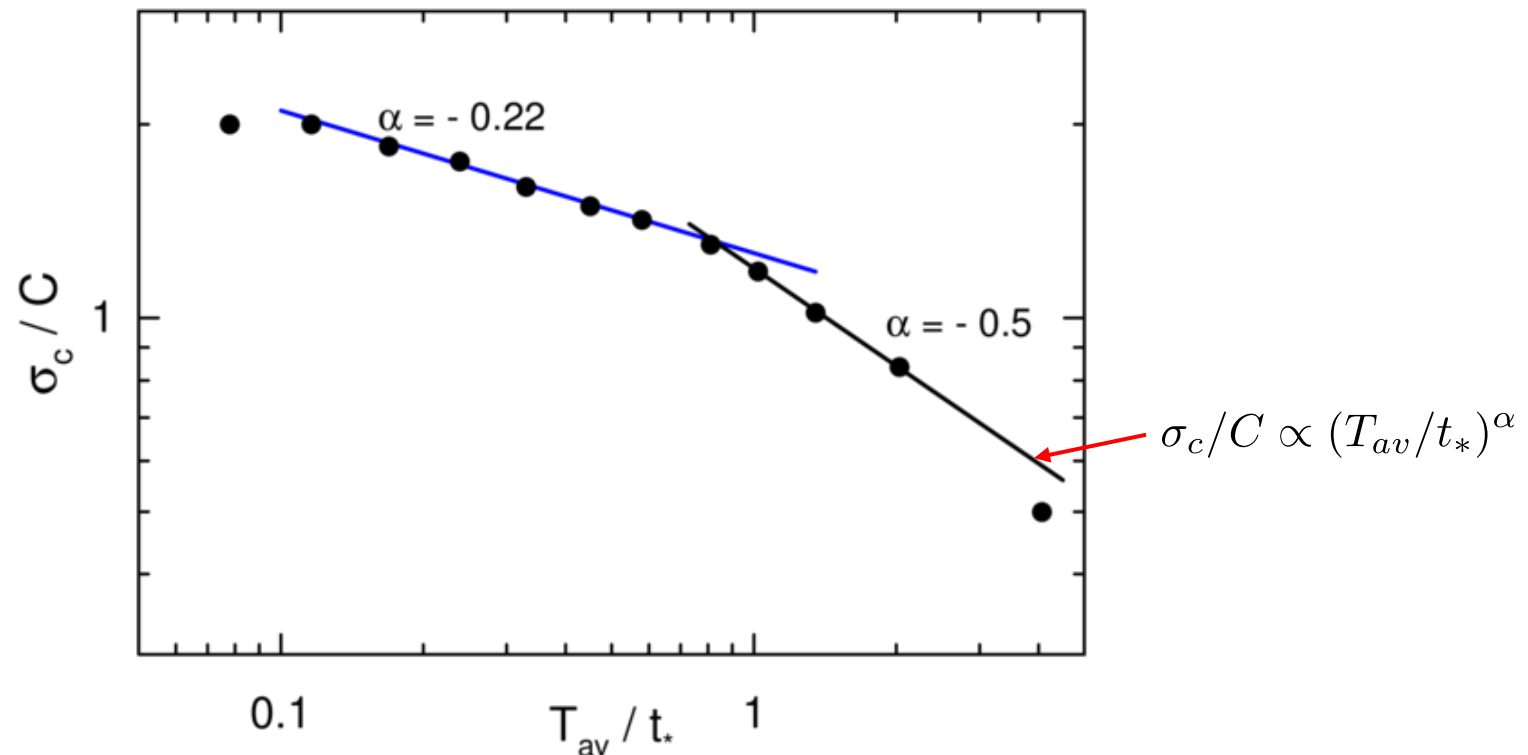
$T_{av} = 1 \text{ min}$   
L2PDM

# Concentration Fluctuation Intensity at Surface: Averaging Time Effect

$$\left(\frac{\sigma_c}{C}\right)^2 = \frac{2(\sigma_{ci}/C)^2}{T_{av}} \cdot \int_0^{T_{av}} (1 - t/T_{av})\rho_c(t)dt \quad (\text{Tennekes \& Lumley, 1972})$$

$$\sigma_c = \sqrt{2}\sigma_{ci} (\tau_c/T_{av})^{1/2} \quad T_{av}/\tau_c \gg 1$$

$$z_s/z_i = 0.07, y = 0, X = 1.55$$



# Concentration Fluctuation Intensity at Surface: Averaging Time ( $T_{av}$ ) Effect

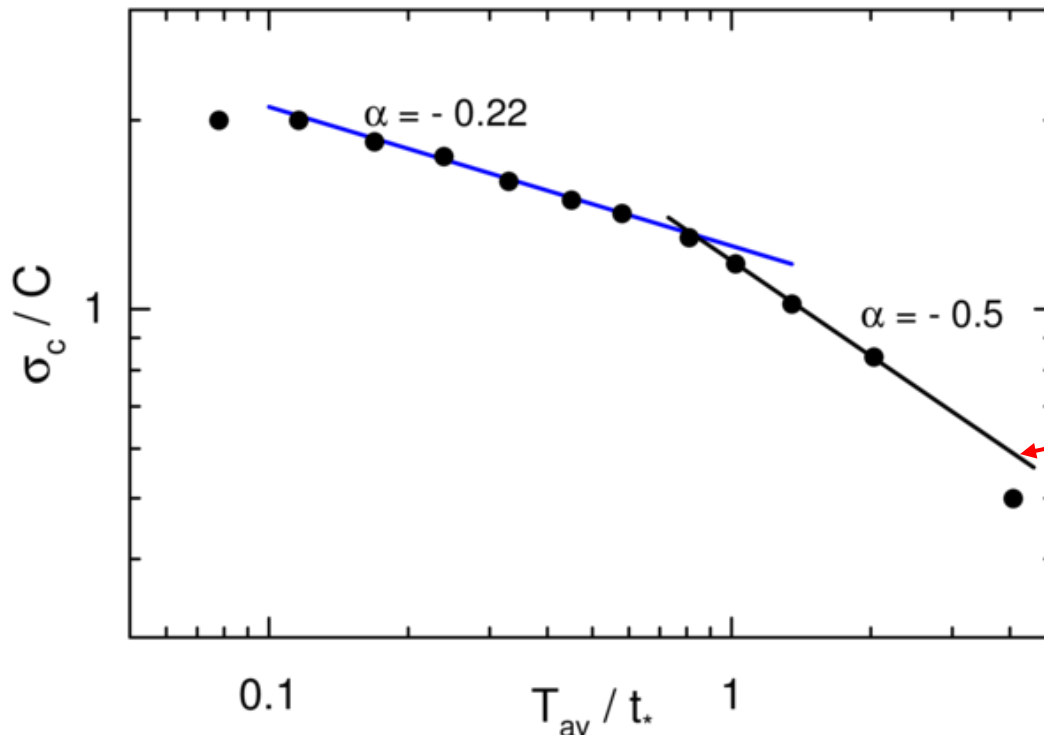
$$\left(\frac{\sigma_c}{C}\right)^2 = \frac{2(\sigma_{ci}/C)^2}{T_{av}} \cdot \int_0^{T_{av}} (1 - t/T_{av}) \rho_c(t) dt \quad (\text{Tennekes \& Lumley, 1972})$$

Concentration autocorrelation

Ensemble  
Variance  
( $T_{av} = 0$ )

$$\sigma_c = \sqrt{2} \sigma_{ci} (\tau_c/T_{av})^{1/2} \quad T_{av}/\tau_c \gg 1$$

$$z_s/z_i = 0.07, \quad y = 0, \quad X = 1.55$$

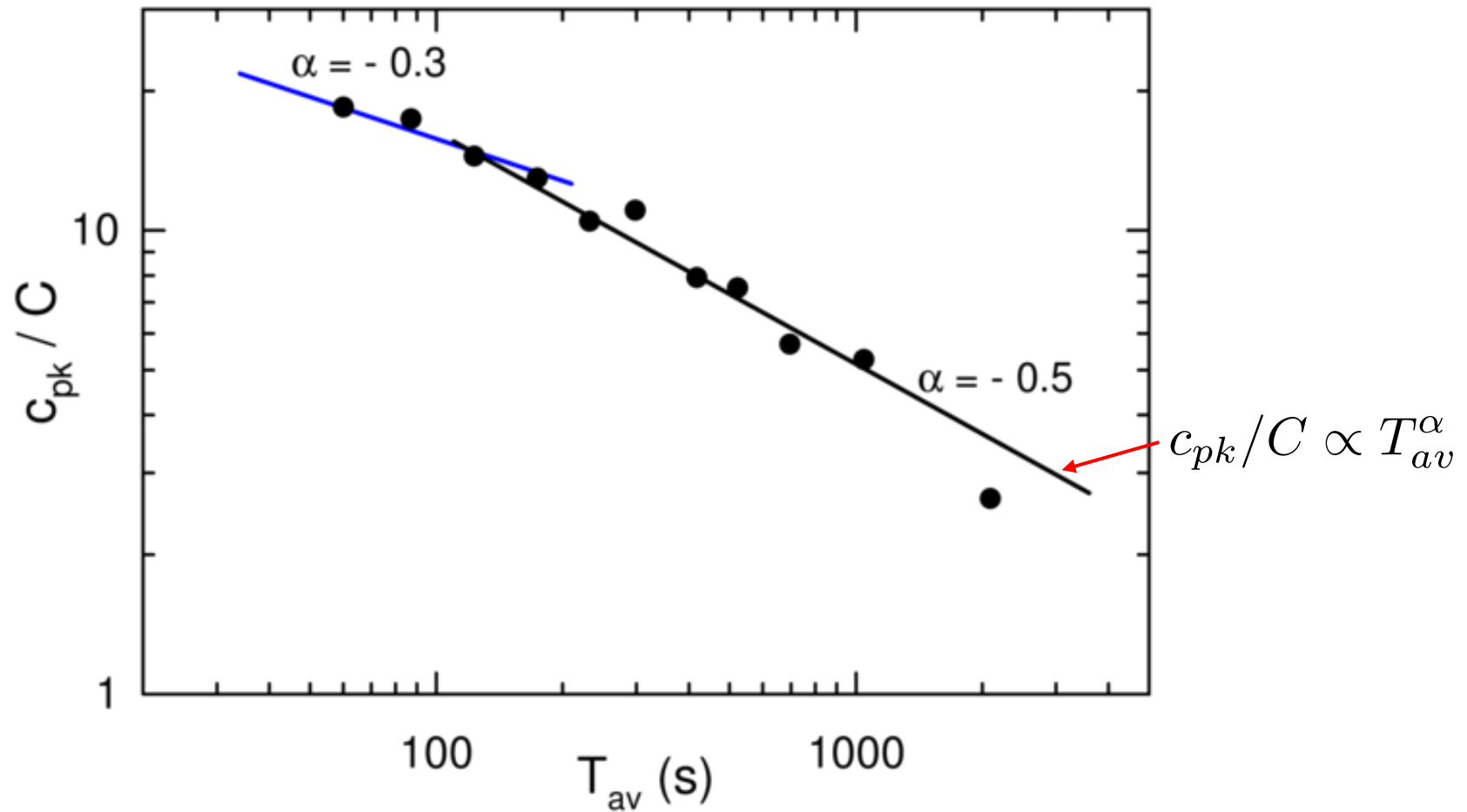


$$\sigma_c/C \propto (T_{av}/t_*)^\alpha$$



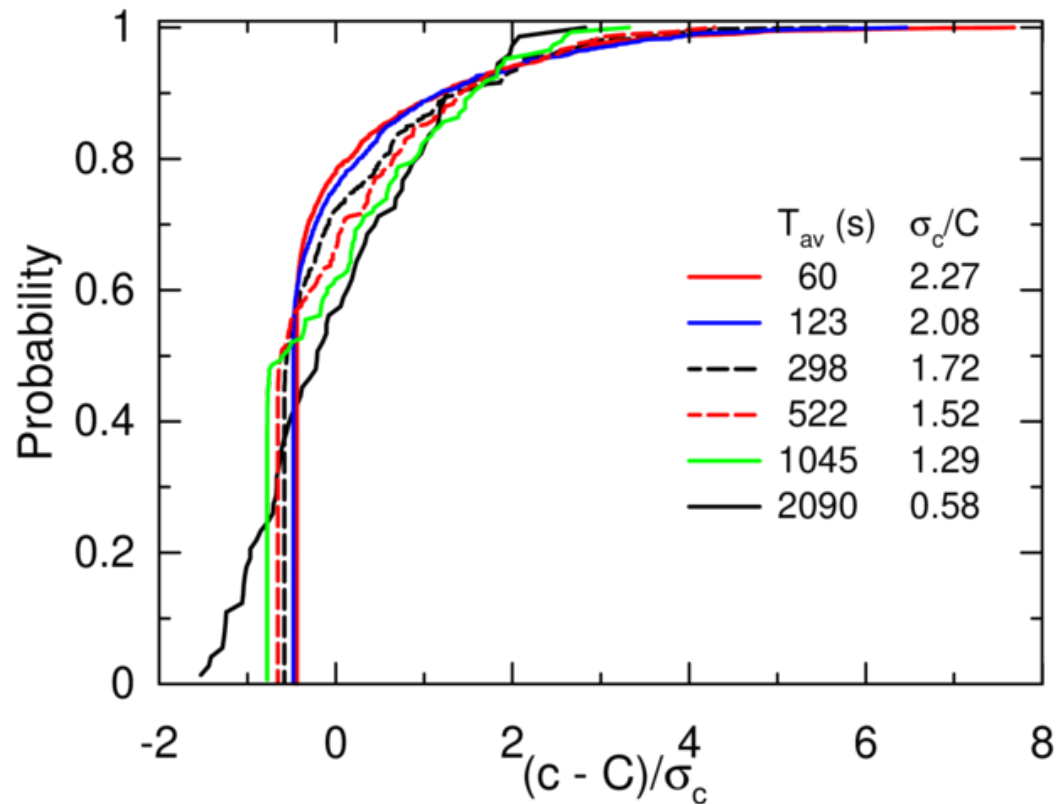
# Peak Surface Concentration as Function of $T_{av}$

$$z_s/z_i = 0.07; y = z = 0; X = X_{mx} = 0.125$$

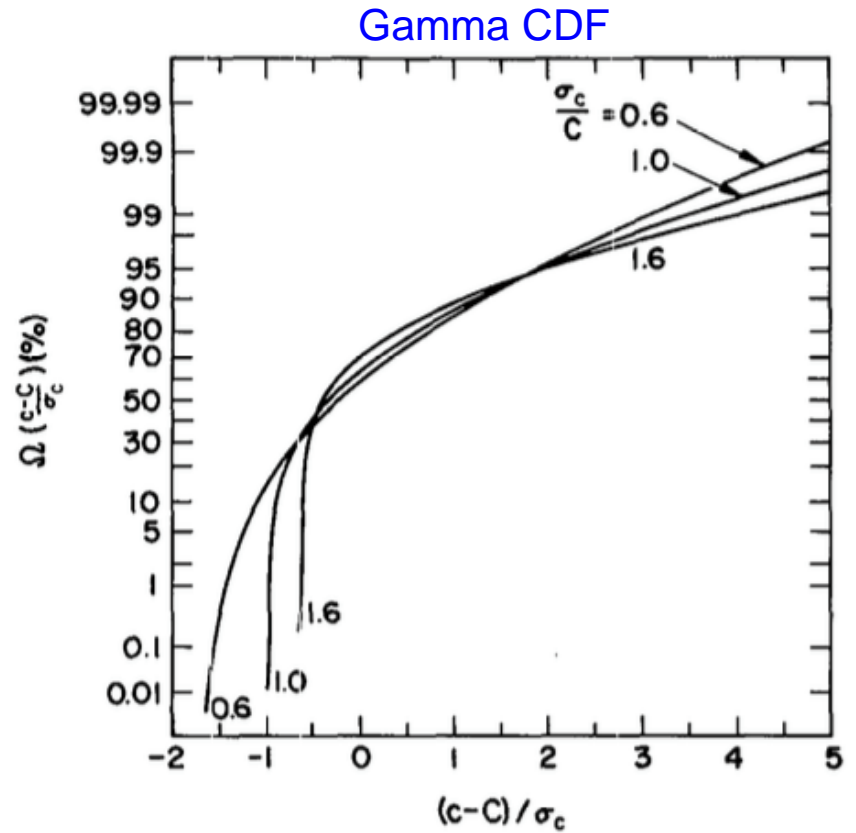
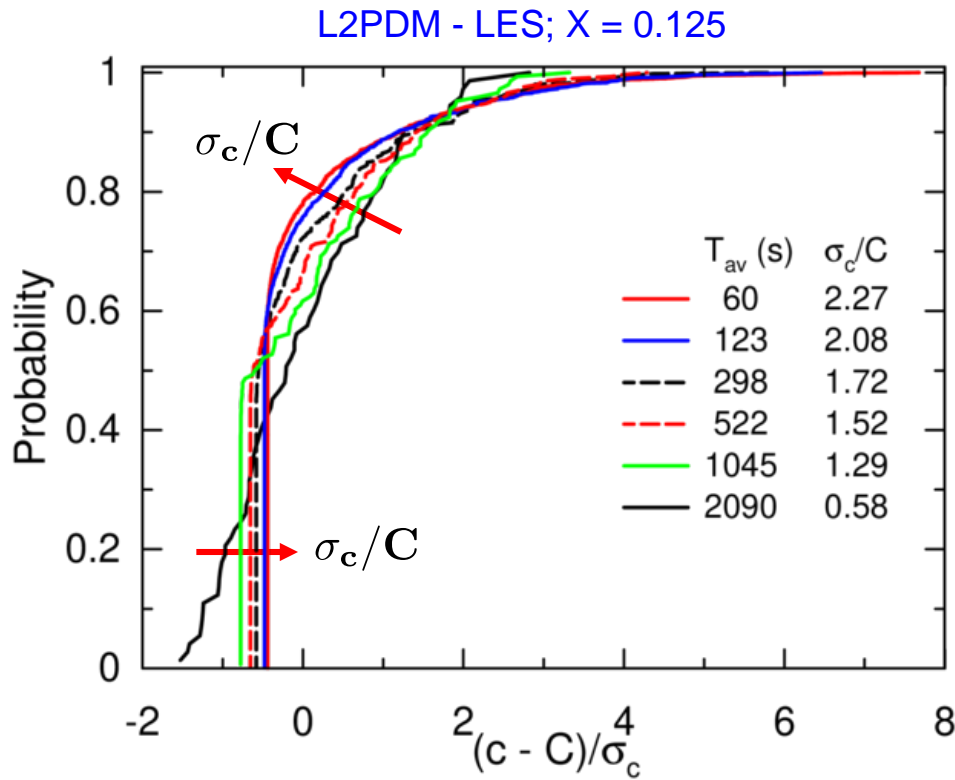


# Cumulative Distribution Function (CDF) of Concentration Fluctuation, $(c - C)/\sigma_c$

$z_s/z_i = 0.07$ ;  $y = z = 0$ ;  $X = X_{mx} = 0.125$



# Cumulative Distribution Function of Concentration Fluctuations



Weil et al. (1992)

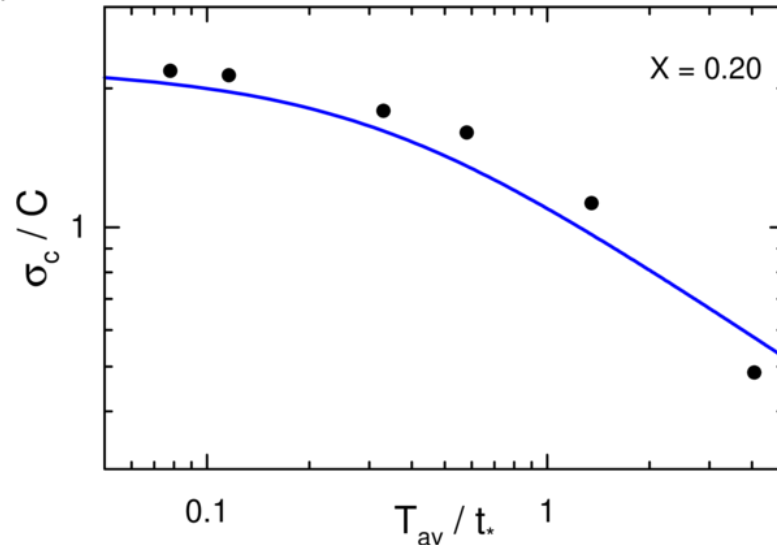
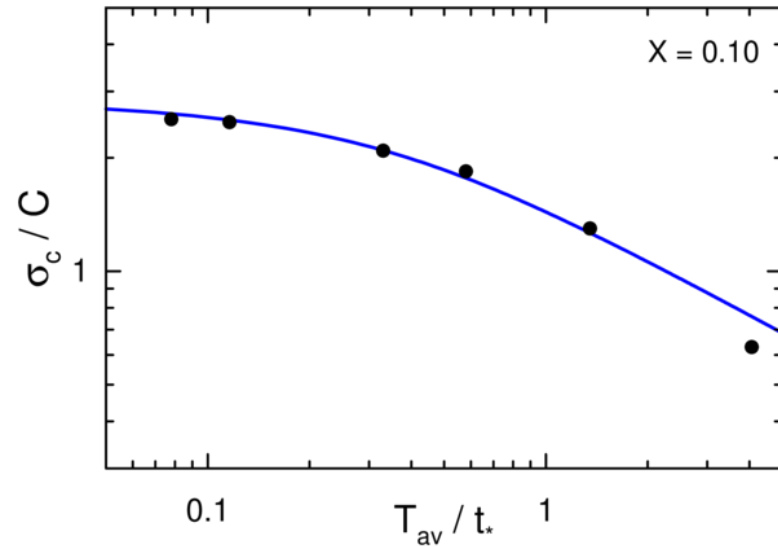
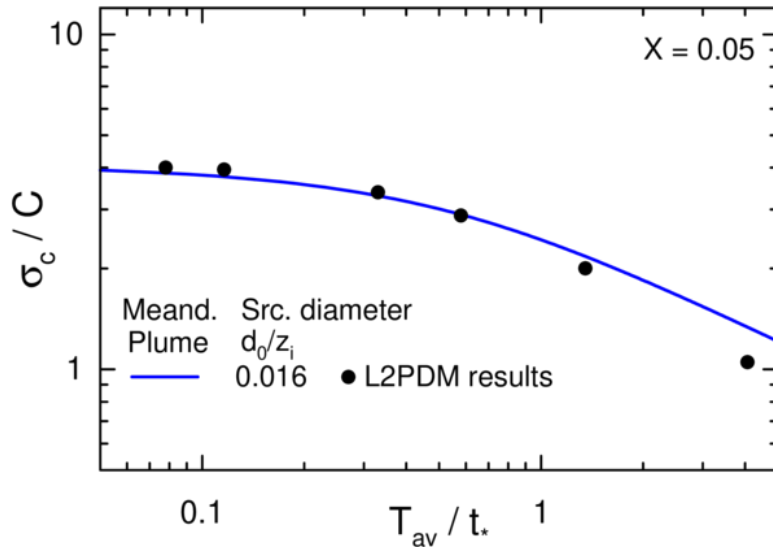
## Applications: Estimate $C_{peak}$

- Parameterized CDF
  - Mean ( $C$ ), Rms ( $\sigma_c$ ), CDF shape
  - Compare L2PDM and Gamma CDFs
- Estimate  $\sigma_c$  with simpler model
  - Lagrangian “1-particle” model (e.g., Manor, 2014; Ferrero et al, 2017, 2019)
  - Extended meandering plume for  $T_{av}$  effects
  - Compare with L2PDM results

# Extended Meandering Plume for $T_{av}$ (Sykes, 1984)

Short times/distances:  $\sigma_i^2 = \sigma_0^2 + \alpha \epsilon t^3$ ,  $\sigma_m = \sigma_{vm} t$

$$z_s/z_i = 0.07; y = z = 0$$



# Summary

- L2PDM matches mean CWIC and surface concentration fields of earlier 1-particle model and laboratory data in a CBL
- Modeled concentration fluctuation intensity agrees with convection tank fluctuation intensity profiles
- L2PDM gives fluctuation intensity, probability distributions of  $c$ , and  $c_{\text{peak}}$  as function of  $T_{\text{av}}$  over 60 s – 2090 s (35 min)
- L2PDM provides data for testing simpler models and applications

## Further Work:

Finer resolution LES (CBL) and greater simulation time

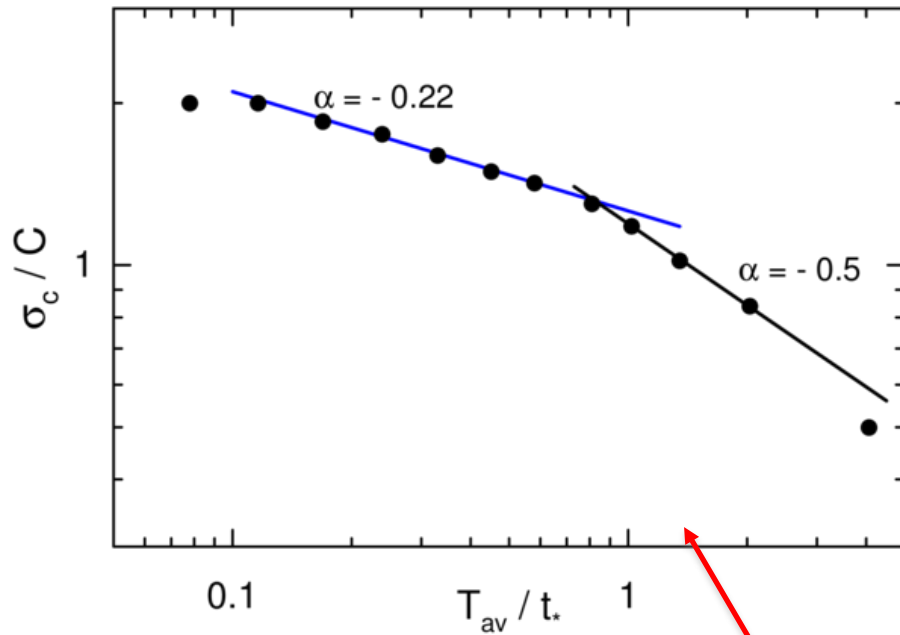
Finer resolution LES and higher wind speeds ( $U/w_* > 1.5$ )

Weakly stable boundary layer

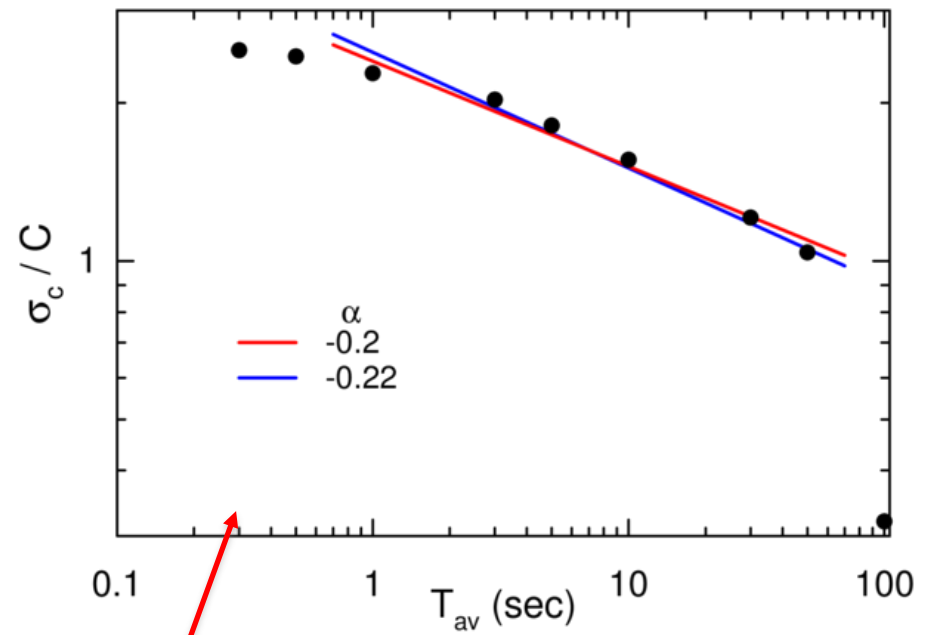


# Concentration Fluctuation Intensity vs $T_{av}$ : Comparison with Field Data

$z_s/z_i = 0.07, y = 0$



Field data: surface source in neutral  
conds.; Mylne & Mason (1991)



Similar slope/dependence on  $T_{av}$  at small  $T_{av}$



# Background

- Importance of statistical variability
  - Estimate peak concentrations—toxics, air quality, odors, chemistry, etc.
  - Need: mean, variance, probability dist.
- Statistics for variable averaging time,  $T_{av}$ 
  - Requires “relative dispersion” of plume
- Approach
  - Lagrangian two-particle dispersion model (L2PDM) driven by large-eddy simulations (LES) of the convective boundary layer (CBL)
- Goal
  - Generate dispersion realizations & statistics for  $T_{av}$ 's: 40 s – 30 min
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  - Provide numerical data for testing simple models

Borex (Denmark, 1992)

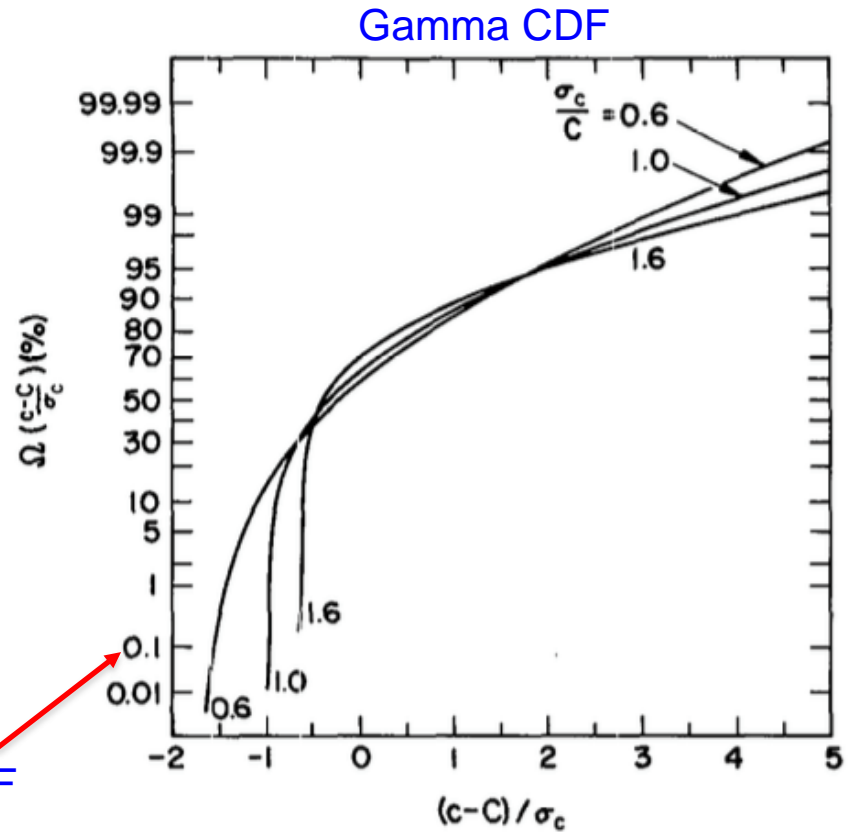
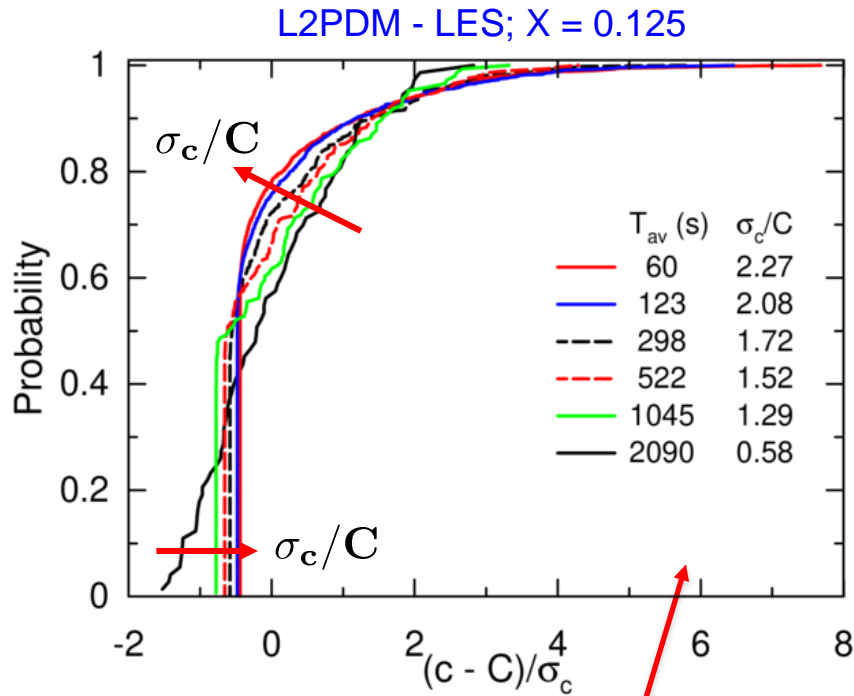


“Passive” or non-buoyant source.

## Applications: Estimate $C$ , $\sigma_c$ , $C_{peak}$

- Parameterized CDF
  - Mean ( $C$ ), Rms ( $\sigma_c$ ), CDF shape
  - Compare L2PDM with Gamma CDF
- Estimate  $\sigma_c$  with simpler model
  - Lagrangian “1-particle” model (e.g., Manor, 2014; Ferrero et al, 2017, 2019)
  - Extended meandering plume for  $T_{av}$  effects

# Cumulative Distribution Function of Concentration Fluctuations



Rescaled L2PDM CDF similar to Gamma CDF

Get similar  $c_{pk}/C$  to Gamma CDF with  $\sigma_c/C = 3.5$

Weil et al. (1992)

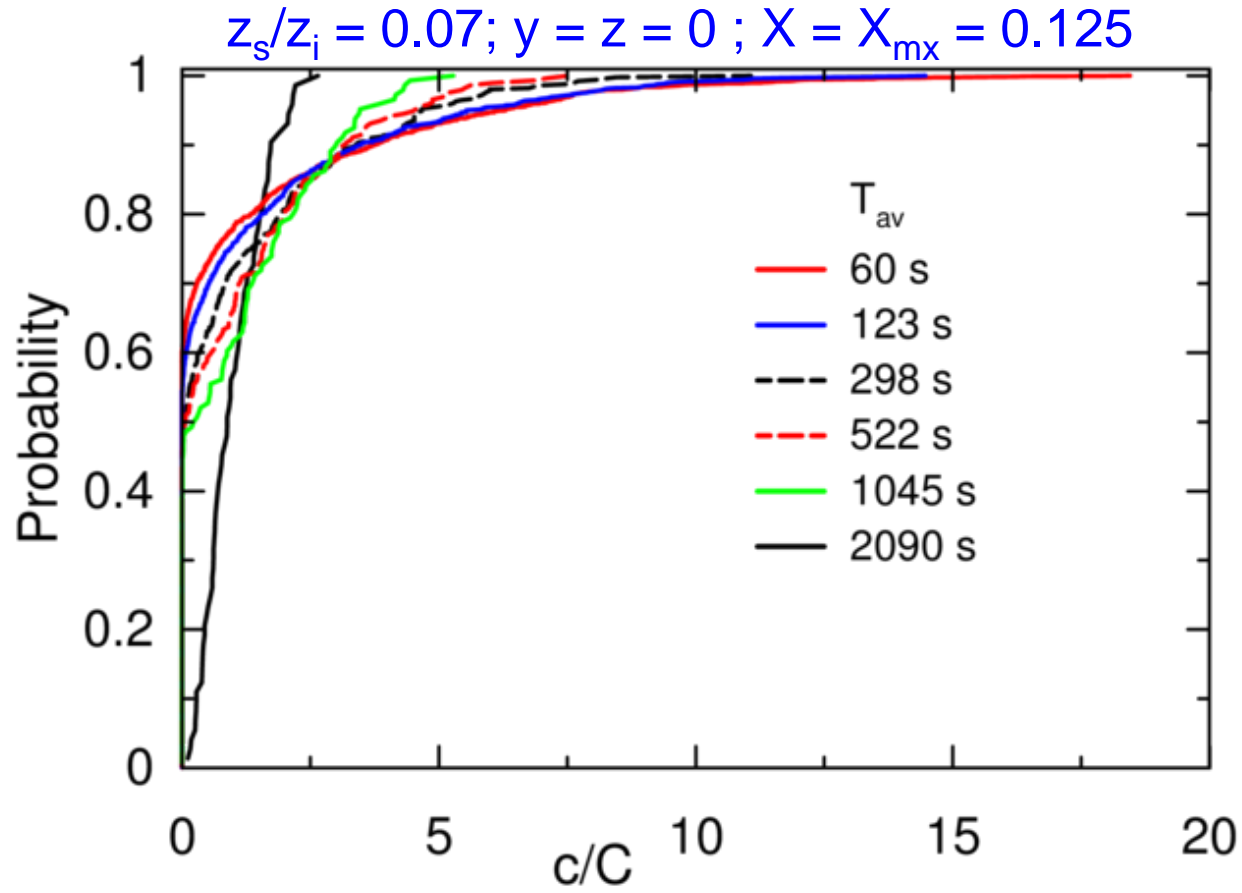
# Summary

- Lagrangian particle & LES approach extended to concentration fluctuations with a 2-particle model (L2PDM)
- L2PDM matches mean CWIC and surface concentration fields of earlier 1-particle model and laboratory data in a CBL
- Modeled concentration fluctuation intensity agrees with convection tank data of vertical and surface fluctuation intensity profiles
- L2PDM provides fluctuation intensity, probability distributions of  $c$ , and  $c_{\text{peak}}$  as function of  $T_{\text{av}}$  over 60 s – 2090 s (35 min)
- L2PDM provides data for testing simpler models and applications

## Further Work:

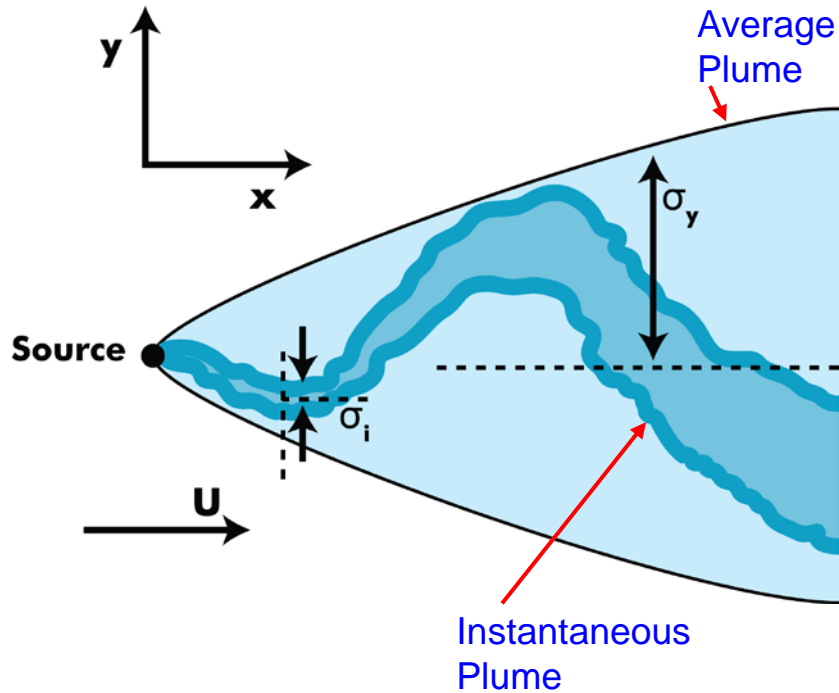
Finer resolution LES (CBL) and greater simulation time  
Finer resolution LES and higher wind speeds ( $U/w_* > 1.5$ )  
Weakly stable boundary layer

# Cumulative Distribution Function / Probability of $c/C$ as Function of $T_{av}$

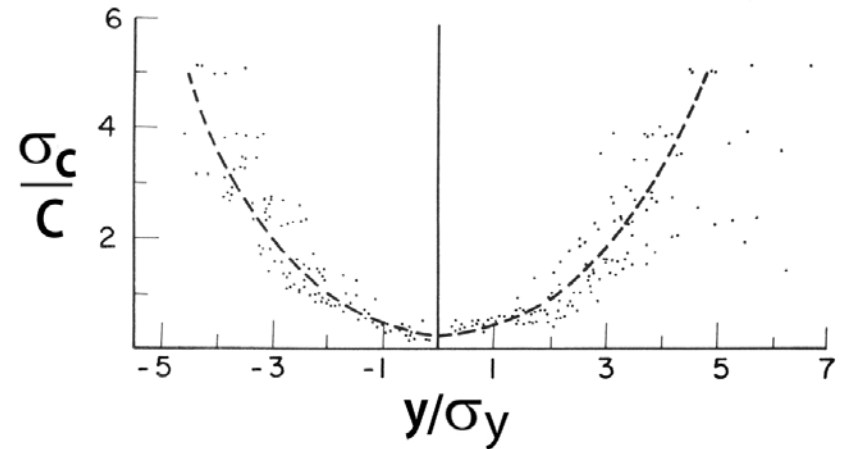


# Generation of Concentration Fluctuations

Meandering Plume Model  
(Gifford 1959)

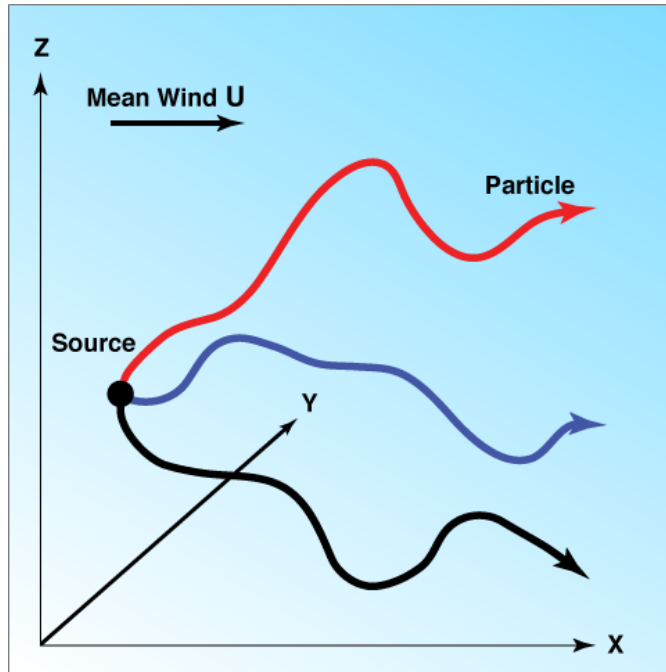


Concentration Fluctuation Intensity  
(Csanady 1973)

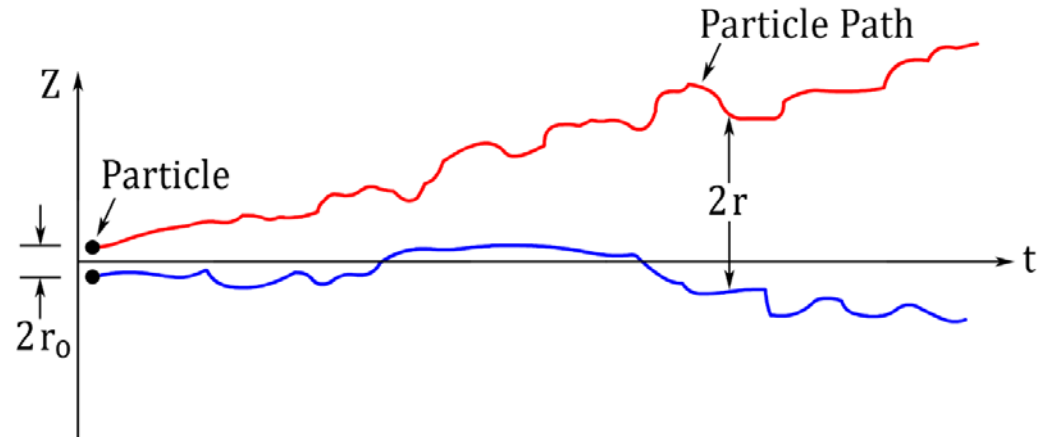


# Lagrangian 1- & 2-Particle Dispersion Models

Applies to 1- & 2-particle models



2-particle model



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$$v_2(\mathbf{x}_{02}, \mathbf{x}_{01}, t) = u_{R2}(\mathbf{x}_{p2}, t) + u_{S2}(\mathbf{x}_{p1}, \mathbf{x}_{p2}, t)$$

$u_{Ri}$  = resolved LES velocity, particle  $i$

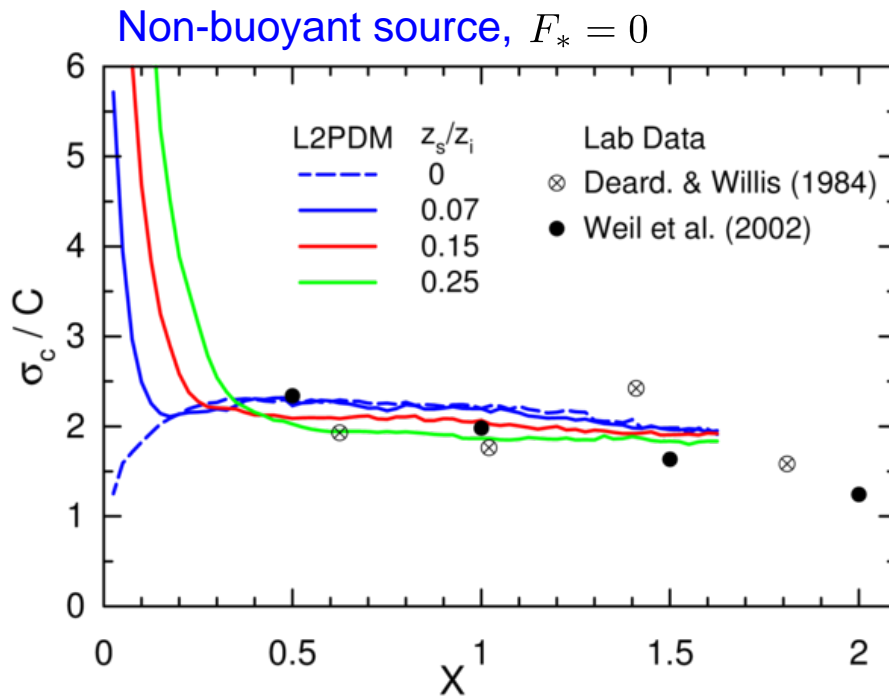
$u_{Si}$  = stochastic subgrid-scale (SGS) velocity

Adopt Thomson's (1990) stochastic model for  $u_{Si}$

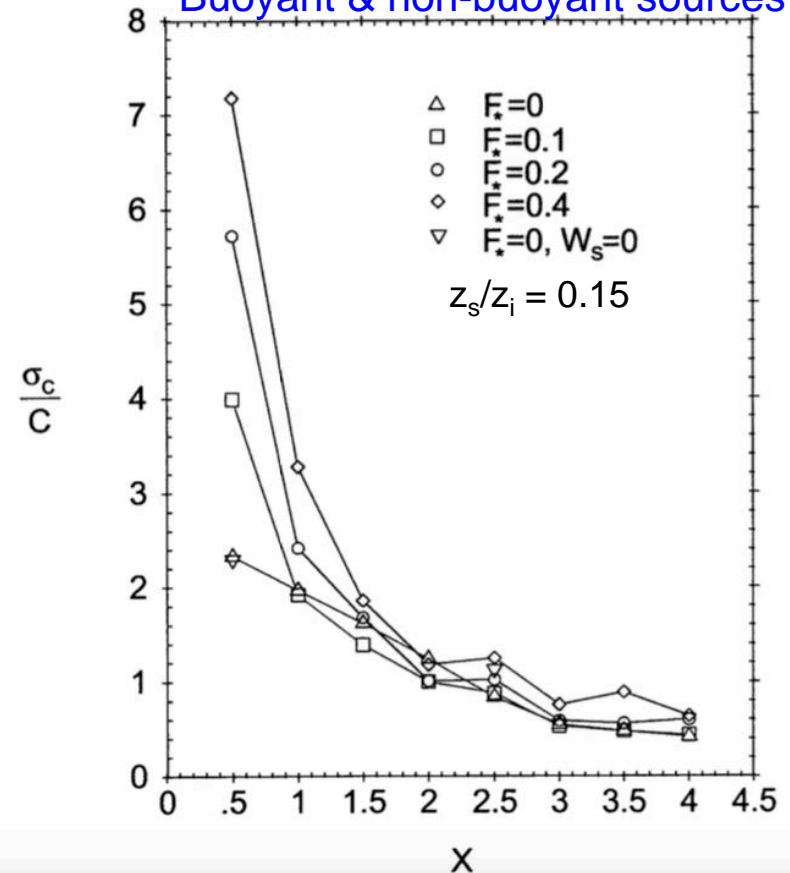
Concentration

$$c(\mathbf{x}, t) = Q \int p_1(\mathbf{x}, t; \mathbf{x}_{S1}, t'_1) dt'_1$$

# Concentration Fluctuation Intensity at Surface: L2PDM vs Lab Data, Multiple Source Heights



Conv. Tank Data (Weil et al., 2002)  
Buoyant & non-buoyant sources





# Peak Surface Concentration as Function of $T_{av}$

$$z_s/z_i = 0.07; y = z = 0; X = X_{mx} = 0.125$$

